Dividing Fractions and Mixed Numbers

BEFORE	▶ Now	WHY?
You multiplied fractions and mixed numbers.	You'll divide fractions and mixed numbers.	So you can set up a slalom skiing course, as in Ex. 35.

📗 **Word Watch**

reciprocal, p. 222

You can use a model to find the quotient $2 \div \frac{1}{4}$.

First draw two unit squares. Then divide each square into fourths.

There are 8 fourths in the model. So, $2 \div \frac{1}{4} = 8$. Notice that there are 4 fourths in each square and there are 2 squares, so $2 \times 4 = 8$. The numbers $\frac{1}{4}$ and 4 are *reciprocals*. Two nonzero numbers whose product is 1 are **reciprocals**.

$\frac{2}{3}$ and $\frac{3}{2}$ are reciprocals, because $\frac{2}{3} \cdot \frac{3}{2} = 1$.

5 and $\frac{1}{5}$ are reciprocals, because $5 \cdot \frac{1}{5} = 1$.

As you saw above, $2 \div \frac{1}{4} = 2 \times 4$, which suggests the following rule.
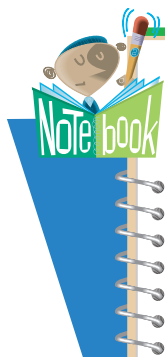
Using Reciprocals to Divide

Words To divide by any nonzero number, multiply by its reciprocal.

Numbers $\frac{3}{4} \div \frac{2}{3} = \frac{3}{4} \cdot \frac{3}{2} = \frac{9}{8}$ **Algebra** $\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c} = \frac{ad}{bc}$

$(b, c, d \neq 0)$

EXAMPLE 1 Dividing a Fraction by a Fraction

$\frac{5}{9} \div \frac{2}{3} = \frac{5}{9} \cdot \frac{3}{2}$ **Multiply by reciprocal.**

$= \frac{5 \cdot \overset{1}{\cancel{3}}}{\underset{3}{\cancel{9}} \cdot 2}$ **Use rule for multiplying fractions.**
Divide out common factor.

$= \frac{5}{6}$ **Multiply.**

EXAMPLE 2 Dividing a Fraction by a Whole Number

$\frac{4}{5} \div 2 = \frac{4}{5} \cdot \frac{1}{2}$ Multiply by reciprocal.

$= \frac{\overset{2}{\cancel{4}} \cdot 1}{5 \cdot \underset{1}{\cancel{2}}}$ Use rule for multiplying fractions.
   Divide out common factor.

$= \frac{2}{5}$ Multiply.

Your turn now   Find the quotient. Simplify if possible.

**1.** $\frac{5}{6} \div \frac{7}{9}$ **2.** $\frac{9}{2} \div \frac{3}{2}$ **3.** $\frac{1}{6} \div 3$ **4.** $\frac{2}{3} \div 4$

EXAMPLE 3 Drawing a Diagram to Solve a Problem

In-line Skating You set up an in-line skating course 21 feet long to practice weaving around cones. You want a cone every $3\frac{1}{2}$ feet, but not at the start or end of the course. How many cones will you need?

Solution

Method 1 Draw a diagram on graph paper. Make the course 21 grid boxes long. Draw a point to mark the location of a cone every $3\frac{1}{2}$ grid boxes.
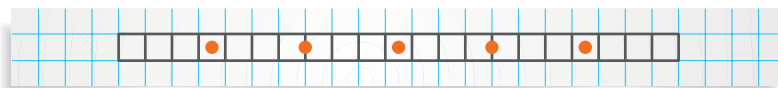
Method 2 Use division.

$21 \div 3\frac{1}{2} = 21 \div \frac{7}{2}$ Write $3\frac{1}{2}$ as an improper fraction.

$= \frac{21}{1} \times \frac{2}{7}$ Multiply by reciprocal.

$= \frac{\overset{3}{\cancel{21}} \times 2}{1 \times \underset{1}{\cancel{7}}}$ Use rule for multiplying fractions.
   Divide out common factor.

$= 6$ Multiply.

The quotient 6 gives you the number of $3\frac{1}{2}$ foot *spaces,* not the number of *cones.* You have to subtract 1 to get the number of cones: $6 - 1 = 5$.

ANSWER You will need 5 cones.

EXAMPLE 4 **Dividing Two Mixed Numbers**

$$8\frac{3}{4} \div 2\frac{5}{8} = \frac{35}{4} \div \frac{21}{8} \qquad \text{Write } 8\frac{3}{4} \text{ and } 2\frac{5}{8} \text{ as improper fractions.}$$

$$= \frac{35}{4} \cdot \frac{8}{21} \qquad \text{Multiply by reciprocal.}$$

$$= \frac{\overset{5}{\cancel{35}} \cdot \overset{2}{\cancel{8}}}{\underset{1}{\cancel{4}} \cdot \underset{3}{\cancel{21}}} \qquad \begin{array}{l}\text{Use rule for multiplying fractions.} \\ \text{Divide out common factors.}\end{array}$$

$$= \frac{10}{3}, \text{ or } 3\frac{1}{3} \qquad \text{Multiply.}$$

Your turn now Find the quotient. Then estimate to check the answer.

5. $3 \div \frac{6}{11}$ **6.** $12 \div 2\frac{2}{5}$ **7.** $5\frac{2}{3} \div \frac{3}{5}$ **8.** $4\frac{1}{2} \div 1\frac{1}{4}$

5.4 Exercises

More Practice, p. 709

Getting Ready to Practice

1. Vocabulary Copy and complete: The reciprocal of $\frac{3}{7}$ is ? .

2. Writing Explain in your own words how to divide a fraction by a fraction.

Write the reciprocal of the number.

3. $\frac{1}{7}$ **4.** 8 **5.** $1\frac{4}{9}$ **6.** $3\frac{3}{4}$

Match the division expression with the related multiplication expression.

7. $\frac{3}{20} \div \frac{4}{9}$ **8.** $\frac{7}{8} \div \frac{3}{13}$ **9.** $1\frac{1}{7} \div 4\frac{1}{3}$ **10.** $6\frac{2}{3} \div 2\frac{1}{4}$

A. $\frac{8}{7} \times \frac{3}{13}$ **B.** $\frac{3}{20} \times \frac{9}{4}$ **C.** $\frac{20}{3} \times \frac{4}{9}$ **D.** $\frac{7}{8} \times \frac{13}{3}$

11. Sandwiches To surprise the guests at a party, the host prepares a $5\frac{1}{2}$ foot long submarine sandwich. The sandwich is cut into 11 equal pieces. How long is each piece?

Practice and Problem Solving

Write the reciprocal of the number.

12. $\frac{5}{6}$ **13.** $1\frac{3}{4}$ **14.** $5\frac{2}{5}$ **15.** 1

Find the quotient. Then check the answer.

16. $\frac{3}{8} \div \frac{1}{4}$ **17.** $\frac{6}{7} \div \frac{5}{14}$ **18.** $\frac{8}{9} \div 1\frac{2}{15}$ **19.** $\frac{7}{12} \div 4$

20. $\frac{9}{10} \div 6$ **21.** $8 \div 2\frac{3}{4}$ **22.** $10 \div 4\frac{1}{6}$ **23.** $4\frac{1}{5} \div \frac{3}{10}$

24. $2\frac{5}{6} \div 7$ **25.** $5 \div \frac{2}{3}$ **26.** $9\frac{4}{5} \div 1\frac{1}{13}$ **27.** $7\frac{1}{6} \div 2\frac{7}{12}$

xy **Algebra** Evaluate the expression when $x = \dfrac{5}{8}$ and $y = 3$.

28. $x \div y$ **29.** $y \div x$ **30.** $4\frac{1}{6} \div y$

31. Cooking Making pasta requires boiling 6 cups of water, but your measuring cup holds only $1\frac{1}{2}$ cups. How many times do you need to fill the measuring cup?

Evaluate the expression.

32. $\left(2\frac{3}{5} \div 3\frac{9}{10}\right) \times 3\frac{5}{8}$ **33.** $\left(5\frac{2}{3} - 3\frac{1}{2}\right) \div 8\frac{3}{5}$ **34.** $\left(6\frac{2}{7} + 3\frac{2}{3}\right) \div 5\frac{3}{4}$

35. Slalom Skiing On a slalom skiing course, the distance from the start to the first gate is 15 meters, and the distance from the last gate to the finish is 20 meters. If the slalom course is 635 meters long and the distance between gates is $1\frac{1}{2}$ meters, how many gates are needed for the course?

36. Draw a Diagram You are an editor for your school yearbook. Each row of photos is $8\frac{5}{8}$ inches wide, including the margins. Each photo is $1\frac{1}{4}$ inches wide, the space between each photo is $\frac{1}{8}$ inch, and each margin is $\frac{1}{4}$ inch. How many photos can fit in one row?

37. Challenge Choose a nonzero number and divide it by a series of fractions, each less than the one before, such as $\frac{3}{4}, \frac{2}{3}, \frac{1}{2}, \ldots$. Describe what happens to the corresponding series of quotients.

Mixed Review

In Exercises 38 and 39, copy and complete the statement using the appropriate metric unit. *(Lesson 2.6)*

38. The capacity of a vase is 325 _?_ .   **39.** A basketball has a mass of 0.62 _?_ .

40. Find the product of $\frac{5}{12}$ and $7\frac{3}{5}$. *(Lesson 5.3)*

Basic Skills **Use a ruler to draw a segment with the given length.**

41. 5 inches   **42.** $3\frac{1}{2}$ inches   **43.** $\frac{3}{4}$ inch   **44.** $2\frac{5}{8}$ inches

Test-Taking Practice

45. **Extended Response** A CD case is $\frac{3}{8}$ inch wide. A cassette case is $\frac{5}{8}$ inch wide. If you want to put 8 cassettes on a shelf that is 20 inches wide, how many CDs would fit in the remaining space? Explain your reasoning.

BRAIN GAME

Mix and Match

Play this game with a partner. Take turns doing the following:

  1. Choose two fractions from the clothesline. (Fractions cannot be used more than once.)

  2. Let one fraction be the dividend and the other be the divisor.

  3. Find the quotient. (Your partner should check your answer.)

  4. Add the quotient to your score. (You both start with a score of zero.)

Once all the fractions have been used, the player with the higher score wins.